NEW EXACT CONDITIONS FOR POSITIVITY
OF QUADRATIC UNCERTAIN POLYNOMIALS

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Abstract

The problem of deriving necessary and sufficient conditions for positivity of a class of uncertain scalar polynomials is considered. The newly obtained result requires a check for positive definiteness of a finite set of matrices, contrary to the classical theorem, which provides an asymptotically exact condition. The suggested approach is illustrated by several examples.

Key words: uncertain quadratic polynomial, Polya’s theorem

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Introduction. This research work deals with the problem for obtaining exact conditions for positivity of a quadratic in the uncertain parameter homogeneous polynomial, defined on the set of nonnegative vectors. There exists a result, which provides an asymptotically exact condition, but its practical application may cause computational difficulties in some more complicated cases. It requires a check for positivity of all coefficients of a homogeneous polynomial of a priori not known “sufficiently large” degree. This fact is the motivation to make an attempt to overcome this shortcoming. Having in mind that any such polynomial can be represented in a quadratic in the uncertain parameter vector compact matrix form, some sufficient conditions for positivity can be easily obtained, expressed in terms of matrix positive definiteness. When exact conditions are required, the main difficulty consists in the fact that there exist some positive coefficients corresponding to the terms $\alpha_i\alpha_j$, $i \neq j$. It is shown that this difficulty can be eliminated when the existing inequalities between some parameters are taken into account. This helps to define a finite set of polynomials. Their positivity is equivalent to the positivity of the given polynomial, and what is more,
this fact can be easily checked. The applicability of this approach is illustrated by several examples.

2. Preliminaries, problem formulation, previous results. The following notations shall be used in the sequel: \( I_k \) is the set of \( k \) integers, \( A = [a_{ij}] \in R_N \) denotes a real \( N \times N \) matrix \( A \) with entries \( a_{ij} \), and \( \alpha \) is a vector with \( N \) entries \( \alpha_i \); \( \Gamma, (m) \) denote the sets of nonnegative vectors \( v \neq 0 \) with respective dimension and the set of \( m \times m \) real matrices with nonpositive off-diagonal entries. Finally, \( \tilde{\alpha}_i \) is a \( N \times N \) vector with all zero entries, except for the \( i \)-th one, which is positive.

Consider the quadratic in the uncertain parameter vector \( \alpha \in \Gamma \) polynomial

\[
p(\alpha) = \sum_{i=1}^{N} c_{ii} \alpha_i^2 + 2 \sum_{j=1, j \neq i}^{N} c_{ij} \alpha_i \alpha_j = \alpha^T C \alpha, \quad C = C^T = [c_{ij}] \in R_N,
\]

with \( 0.5 N(N + 1) \) coefficients. It is desired to obtain exact conditions under which \( (1) \) is positive on \( \Gamma \).

Recall a classical result which provides an asymptotically exact condition for the solution of this problem.

**Theorem 2.1** [1]. Let a given polynomial in \( (1) \) be positive definite. There exists some sufficiently large integer \( d^* \), such that for \( d \geq d^* \), all coefficients of the homogeneous polynomial \( \bar{p}(\alpha) = (\alpha_1 + \alpha_2 + \cdots + \alpha_N)^d p(\alpha) \) of degree \( d + 2 \) are positive.

Theorem 2.1 is known as the Polya’s theorem. It has many important applications, e.g., in studying positive definiteness of homogeneous uncertain polynomials [5, 7], robust stability analysis of uncertain systems [4, 6], etc. The main difficulty, from a computational point of view, always arises from the simple fact that the exact value of \( d^* \) is a priori unknown. This research is an attempt to suggest an approach, which also leads to an exact solution, but requires a known finite number of computational procedures. Before that, we need a well known important result concerning matrices belonging to the set \( L(m) \).

**Theorem 2.2** [2, 3]. Let \( M \in L(m) \). There exists an eigenvector \( v \in \Gamma \), \( v \in R_m \), for matrix \( M \), corresponding to a real eigenvalue \( \lambda_m \leq Re\lambda_i \), \( i = 1, 2, \ldots, m - 1 \), i.e., \( Mv = \lambda_m v \).

3. Main results. Let \( C(k) \) be a matrix obtained by deleting \( k \) columns \( c_i \) and rows \( r_i = c_i^T \), \( i \in I_k \), from \( C \) in \( (1) \), which contain only nonnegative off-diagonal entries. Denote these columns \( c_i^+ \). Note that if \( k = N - 1 \), then, due to the symmetry of \( C \), all its off-diagonal entries are nonnegative, i.e., \( c_i = c_i^+ \), \( i = 1, \ldots, N \). Consider now three different special cases in which respective simple exact conditions are easily obtained.

**Lemma 3.1.** Let (i) \( c_i = c_i^+ \), \( i = 1, \ldots, N \), or (ii) \( C \in L(N) \), or (iii) \( c_i = c_i^+ \), \( i \in I_k \), \( 0 < k \leq N - 2 \) and \( C(k) \in L(N - k) \). Then, \( p(\alpha) > 0 \), \( \alpha \in \Gamma \), if and only if

(i) \( c_{ii} > 0 \), \( i = 1, \ldots, N \),  
(ii) \( C > 0 \),  
(iii) \( C(k) > 0 \), \( c_{ii} > 0 \), \( c_{ii} \in c_i^+ \), \( i \in I_k \).
Proof. Let \( p(\alpha) > 0, \alpha \in \Gamma \).

Case (i): Since \( p(\tilde{\alpha}_i) = c_{ii}\tilde{\alpha}_i^2 > 0, i = 1, \ldots, N \Rightarrow c_{ii} > 0, i = 1, \ldots, N \).

Case (ii): In accordance with Theorem 2.2, there exists a vector \( \alpha_N \in \Gamma \), such that \( C\alpha_N = \lambda_N(C)\alpha_N, \lambda_N(C) \leq \lambda_i(C), i = 1, \ldots, N - 1 \Rightarrow p(\alpha_N) = \lambda_N(C)\alpha_N^T\alpha_N > 0 \Rightarrow C > 0 \).

Case (iii): \( p(\alpha) \) in (1) can be represented as \( p(\alpha) = p_1(\tilde{\alpha}) + p_2(\alpha) \), where

\[
p_1(\tilde{\alpha}) = \tilde{\alpha}_T C(k) \tilde{\alpha}, \quad C(k) \in L(N - k),
\]

\[
p_2(\alpha) = \sum_{i \in I_k} \alpha_i \left( c_{ii} \alpha_i + 2 \sum_{j=1, j \neq i}^{N} c_{ij} \alpha_j \right), \quad c_{ij} \in (c_i)^+, i \in I_k
\]

and since \( \alpha_i \notin \tilde{\alpha}, i \in I_k \). Therefore, \( \tilde{\alpha} = 0 \) and \( \alpha_i = 0, i \in I_k \), are two admissible different cases, which result in \( p(\alpha) = p_2(\alpha) > 0 \) and \( p(\alpha) = p_1(\tilde{\alpha}) > 0 \), respectively. Since \( p_2(\tilde{\alpha}_i) = c_{ii}\tilde{\alpha}_i^2 > 0, i \in I_k \) and \( p_1(\tilde{\alpha}_{N-k}) = \lambda_{N-k}[C(k)]\tilde{\alpha}_{N-k}^T\tilde{\alpha}_{N-k} > 0, \alpha_{N-k} \in \Gamma(N-k) \), where \( \alpha_{N-k} \) is an eigenvector corresponding to the minimal eigenvalue of \( C(k) \), are also admissible cases, then, statement (iii) must hold by necessity. This proves the necessity part of the Lemma.

Suppose that statements (i)–(iii) hold. In case (i), \( p(\alpha) \geq \sum_{i=1}^{N} c_{ii} \alpha_i^2 \geq \min c_{ii} \alpha^T \alpha > 0, \alpha \in \Gamma \).

In case (ii), \( C > 0 \) always means positivity of (1) for all vectors \( \alpha \). Consider the estimate

\[
p(\alpha) \geq \lambda_{N-k}[C(k)]\tilde{\alpha}_{N-k}^T\alpha_{N-k} + \sum_{i \in I_k} c_{ii} \alpha_i^2 \geq \min \{ \lambda_{N-k}[C(k)], \min c_{ii} \} \alpha^T \alpha > 0.
\]

This completes the proof of the sufficiency part of the Lemma. \( \square \)

The main difficulty lies in the fact that none of the above made supposition holds in the general case, i.e., matrix \( C \) in (1) has an arbitrary structure and some of its off-diagonal entries are positive. In this case, a new solution to the problem is the main contribution of this research. Before that, we present a result which plays an essential role in its derivation.

Lemma 3.2. Consider a set of \( 2t, t \geq 1 \), polynomials \( p_s(\alpha), \alpha \in \Gamma \), and suppose that there exist \( 2t \) subsets \( \Gamma_s \), such that the following \( t \) pairs of inequalities hold:

\[
(2) \quad p_s(\alpha) \leq p_{s+1}(\alpha), \alpha \in \Gamma_s, p_{s+1}(\alpha) < p_s(\alpha), \alpha \in \Gamma_{s+1}; \quad \Gamma_s \cup \Gamma_{s+1} = \Gamma, s = 1, 3, \ldots, 2t - 1.
\]

Then,

\[
p_s(\alpha) > 0, \alpha \in \Gamma_s, s = 1, 2, \ldots, 2t \Leftrightarrow p_s(\alpha) > 0, \alpha \in \Gamma, s = 1, 2, \ldots, 2t - 1.
\]
Proof. Let \( p_s(\alpha) > 0, \alpha \in \Gamma_s, s = 1, 2, \ldots, 2t \). Having in mind the inequalities (2), one gets

\[
\begin{align*}
\begin{cases}
p_s(\alpha) > 0, \alpha \in \Gamma_s \\
p_{s+1}(\alpha) > 0, \alpha \in \Gamma_{s+1}
\end{cases}
\Rightarrow \begin{cases}
p_s(\alpha) > 0, \alpha \in \Gamma_s \\
p_s(\alpha) > 0, \alpha \in \Gamma_{s+1}
\end{cases}
\Rightarrow \begin{cases}
p_s(\alpha) > 0, \alpha \in \Gamma_s \cup \Gamma_{s+1} \equiv \Gamma \\
p_{s+1}(\alpha) > 0, \alpha \in \Gamma_{s+1} \cup \Gamma_s \equiv \Gamma
\end{cases}
\]

which proves the sufficiency part. If \( p_s(\alpha) > 0, \alpha \in \Gamma, s = 1, 2, \ldots, 2t \), then, obviously, these polynomials are positive on the respective subsets as well. This completes the proof of the necessity part.

Let \( \varphi, 0 < \varphi < 0.5N(N - 1) \), denote the number of positive polynomial coefficients \( c_{ij}, i < j \), in (1). Then any term \( c_{ij}\alpha_i\alpha_j, c_{ij} > 0, i < j \), can be bounded as follows:

\[
c_{ij}\alpha_i^2 \leq c_{ij}\alpha_i\alpha_j \leq c_{ij}\alpha_j^2, \text{ if } \alpha_i \leq \alpha_j,
\]

\[
c_{ij}\alpha_j^2 \leq c_{ij}\alpha_i\alpha_j \leq c_{ij}\alpha_i^2, \text{ if } \alpha_i > \alpha_j.
\]

In a similar way, respective lower and upper bounds can be obtained for all \( \varphi \) terms. All possible inequality combinations between the parameters \( \alpha_i \) and \( \alpha_j \), \( c_{ij} > 0 \), are exactly \( 2t = 2^\varphi \). What is more, they determine the respective \( 2t \) subsets \( \Gamma_s \), on which the following polynomials are defined:

(3) \[
p_s(\alpha) = \sum_{i=1}^{N} c_{ii,s} \alpha_i^2 + 2 \sum_{j=1, j>i}^{N} c_{ij}\alpha_i\alpha_j, c_{ij} \leq 0, j > i, \alpha \in \Gamma_s, s = 1, 2, \ldots, 2t,
\]

where

\[
c_{ii,s} = c_{ii} + \sum_{j=1, j>i}^{N} c_{ij}, \text{ if } c_{ij} > 0 \text{ and } \alpha_i \leq \alpha_j,
\]

\[
c_{ii,s+1} = c_{ii} + \sum_{j=1, j>i}^{N} c_{ij}, \text{ if } c_{ij} > 0 \text{ and } \alpha_i > \alpha_j.
\]

From this definition, it follows that the \( 2t \) polynomials in (3) satisfy the inequalities in (2) and, in addition, they serve as lower and upper bounds for (1) on the respective subsets \( \Gamma_s \), i.e.,

\[
p_s(\alpha) \leq p(\alpha) \leq p_{s+1}(\alpha), \alpha \in \Gamma_s,
\]

\[
p_{s+1}(\alpha) \leq p(\alpha) < p_s(\alpha), \alpha \in \Gamma_{s+1},
\]
and $\Gamma_s \cup \Gamma_{s+1} \equiv \Gamma$, $s = 1, 3, \ldots, 2t - 1$. The polynomials (3) can be represented in matrix form as

$$p_s(\alpha) = \alpha^T C_s \alpha, \quad C_s \in L(N), \quad \alpha \in \Gamma_s, \quad s = 1, 2, \ldots, 2^\bar{\phi}.$$ 

By making extensive use of Lemma 3.2, the next result provides an exact condition for positivity of the considered class of uncertain polynomials in (1).

**Lemma 3.3.** The polynomial (1) is positive, if and only if

$$C_s > 0, \quad s = 1, 2, \ldots, 2^\bar{\phi}.$$

**Proof.** Let all matrices be positive definite and consider an arbitrary given vector $\alpha_g \in \Gamma$. There exists some subset $\Gamma_s$, such that $\alpha_g \in \Gamma_s$, $0 < p_s(\alpha_g) \leq p(\alpha_g)$. Since the parameter vector is arbitrarily chosen, it follows that (1) is positive on the whole set $\Gamma$. This proves the sufficiency part.

Suppose that polynomial (1) is positive on $\Gamma$. It follows that the polynomials (3) are also positive on the respective subsets. But this is possible if and only if they are positive on $\Gamma$, in accordance with Lemma 2. Since $C_s \in L(N)$, $s = 1, 2, \ldots, 2^\bar{\phi}$, this is equivalent to the positive definiteness in all of them. This completes the proof of the necessity part.

The exact condition for positivity of this class of uncertain scalar polynomials is obtained as a condition for positive definiteness of a finite set of matrices which allows to evaluate the computational complexity a priori.

The next result generalizes Lemma 3.2, statement (iii) and Lemma 3.3.

**Lemma 3.4.** Let $k$, $0 \leq k \leq N - 3$, columns of $C$ contain only nonnegative off-diagonal entries, i.e., $c_{ii} = c_i^+$, $i \in I_k$, but $C(k) \notin L(N - k)$. If $\bar{\varphi} < 0.5(N - k)(N - k - 1)$, denotes the number of positive off-diagonal entries of $C(k)$, then the polynomial in (1) is positive on $\Gamma$ if and only if $c_{ii} > 0$, $i \in I_k$ and all respective matrices $C_s(k)$, $s = 1, 2, \ldots, 2^\bar{\phi}$ are positive definite.

**Proof.** It follows from the proof of Lemmas 3.2 and 3.3. If $k = 0$, then $C(0) = C$, $\bar{\varphi} = \varphi$, $C_s(k) = C_s$, and this is the case considered in Lemma 3.3.

Let $k > 0$. Using the notations in the proof of Lemma 3.3, statement (iii), recall the representation of (1) as a sum of two polynomials, i.e., $p(\alpha) = p_1(\bar{\alpha}) + p_2(\alpha)$. It was shown that each of them must be positive. The second term is always nonnegative and its positivity is guaranteed by the first part of the statement of this Lemma. Since the coefficient matrix $C(k) \notin L(N - k)$, one has to apply Lemma 3 for the polynomial $p_1(\bar{\alpha})$, in order to get an exact solution condition for this most general case. As it was already explained above, this is equivalent to the positive definiteness of the respective matrices $C_s(k)$, $s = 1, 2, \ldots, 2^\bar{\phi}$. This completes the proof.

Lemma 3.4 not only generalizes the previous results, but it helps to decrease the computational complexity, required by this approach, in a case, when $k > 0$.
The applicability of the main result will be illustrated by examples.

4. Examples. Consider a polynomial (1) with \( N = 3 \), and suppose that \( c_{12} > 0, c_{13} > 0, c_{23} < 0 \). Then, the respective four polynomials are given by

\[
p_1(\alpha) = (c_{11} + 2c_{12} + 2c_{13})\alpha_1^2 + c_{22}\alpha_2^2 + 2c_{23}\alpha_2\alpha_3, \quad \Gamma_1 \equiv \{\alpha_1 \leq \alpha_2, \alpha_1 \leq \alpha_3\}
\]

\[
p_2(\alpha) = c_{11}\alpha_1^2 + (c_{22} + 2c_{12})\alpha_2^2 + (c_{33} + 2c_{13})\alpha_3^2 + 2c_{23}\alpha_2\alpha_3, \quad \Gamma_2 \equiv \{\alpha_1 > \alpha_2, \alpha_1 > \alpha_3\}
\]

\[
p_3(\alpha) = (c_{11} + 2c_{12})\alpha_1^2 + c_{22}\alpha_2^2 + (c_{33} + 2c_{13})\alpha_3^2 + 2c_{23}\alpha_2\alpha_3, \quad \Gamma_3 \equiv \{\alpha_1 \leq \alpha_2, \alpha_1 > \alpha_3\}
\]

\[
p_4(\alpha) = (c_{11} + 2c_{13})\alpha_1^2 + (c_{22} + 2c_{12})\alpha_2^2 + c_{33}\alpha_3^2 + 2c_{23}\alpha_2\alpha_3, \quad \Gamma_4 \equiv \{\alpha_1 > \alpha_2, \alpha_1 \leq \alpha_3\}.
\]

One can easily verify that these polynomials satisfy the inequalities (2). It is interesting to see whether Lemma 3.3 confirms statement \((iii)\) of Lemma 3.2, since the polynomial in this example satisfies case \((iii)\). Note that \( c_1 = c_1^+ \), i.e., \( k = 1 \). In accordance with Lemma 3.2, statement \((iii)\), this polynomial is positive if and only if \( c_{11} > 0 \) and the respective \( 2 \times 2 \) matrix \( C(1) \) is positive, which is equivalent to \( c_{22}c_{33} > c_{23}^2 \). The four coefficient matrices in this case are

\[
C_1 = \begin{bmatrix}
c_{11,1} & 0 & 0 \\
0 & c_{22} & c_{23} \\
0 & c_{23} & c_{33}
\end{bmatrix}, \quad C_2 = \begin{bmatrix}
c_{11} & 0 & 0 \\
0 & c_{22,2} & c_{23} \\
0 & c_{23} & c_{33,2}
\end{bmatrix},
\]

\[
C_3 = \begin{bmatrix}
c_{11,3} & 0 & 0 \\
0 & c_{22} & c_{23} \\
0 & c_{23} & c_{33,3}
\end{bmatrix}, \quad C_4 = \begin{bmatrix}
c_{11,4} & 0 & 0 \\
0 & c_{22,4} & c_{23} \\
0 & c_{23} & c_{33}
\end{bmatrix}.
\]

Remember that \( c_{ii,s} \geq c_{ii}, \forall i, s \). The two conditions are \( c_{11} > 0 \), obtained from matrix \( C_2 \) and \( c_{22}c_{33} > c_{23}^2 \), obtained from matrix \( C_1 \), which guarantee positive definiteness of all four matrices.

Consider a polynomial (1) with four uncertain parameters

\[
p(\alpha) = 2\alpha_1^2 + 3\alpha_2^2 + \alpha_3^2 + 5\alpha_1^2 - 2\alpha_1\alpha_2 + 7\alpha_1\alpha_3 - 3.6\alpha_1\alpha_4 - 0.4\alpha_2\alpha_3 - 4\alpha_2\alpha_4 - 0.2\alpha_3\alpha_4
\]

and matrix \( C \), given by

\[
C = \begin{bmatrix}
2 & -1 & 3.5 & -1.8 \\
-1 & 3 & -0.2 & -2 \\
3.5 & -0.2 & 1 & -0.1 \\
-1.8 & -2 & -0.1 & 5
\end{bmatrix}, \quad \lambda_{\min}(C) = -2.362.
\]

Since \( C \) is not a positive definite matrix, no conclusion can be made, since \( \alpha \) is a nonnegative vector and \( C > 0 \) is only a sufficient condition. The term \( 7\alpha_1\alpha_3 \geq 0 \) is not true for all \( \alpha \), and the conclusion

\[
\alpha \geq 0 \quad \text{and} \quad C > 0
\]

provides a necessary condition for the polynomial to be positive. Since \( c_{22}c_{33} > c_{23}^2 \) is satisfied, \( \alpha_2 \) and \( \alpha_3 \) can be chosen such that \( p(\alpha) \) becomes positive.
0, \alpha \in \Gamma$ and therefore one possible way to study positivity is to check out positive definiteness of a matrix $C_0 = [c_{0,ij}]$, $c_{0,ij} = c_{ij}$, except for entries $c_{0,13} = c_{0,31} = 0$. This is equivalent to consider a lower polynomial bound $\alpha^T C_0 \alpha \leq \alpha^T C \alpha$, $\alpha \in \Gamma$, but it is clear that if $C_0$ is positive definite, this is again only a sufficient condition for positivity of (1). Unfortunately, $\lambda_{\text{min}}(C_0) = -0.0204$ and again no conclusion can be drawn. Application of Lemma 3.3 is illustrated next. The two matrices corresponding to the respective subsets are given below

$$C_1 = \begin{bmatrix}
9 & -1 & 0 & -1.8 \\
-1 & 3 & -0.2 & -2 \\
0 & -0.2 & 1 & -0.1 \\
-1.8 & -2 & -0.1 & 5
\end{bmatrix} \text{ if } \alpha_1 \leq \alpha \Rightarrow c_{11,1} = c_{11} + 2c_{13},$$

$$C_2 = \begin{bmatrix}
2 & -1 & 0 & -1.8 \\
-1 & 3 & -0.2 & -2 \\
0 & -0.2 & 8 & -0.1 \\
-1.8 & -2 & -0.1 & 5
\end{bmatrix} \text{ if } \alpha_1 > \alpha_3 \Rightarrow c_{33,2} = c_{33} + 2c_{13}.$$  

Computations show that $\lambda_{\text{min}}(C_1) = 0.8935$, $\lambda_{\text{min}}(C_2) = 0.0005$, i.e., both matrices are positive definite and therefore the given polynomial is positive on the uncertainty set, in accordance with Lemma 3.3.

The third example illustrates the applicability of Lemma 3.4. Consider a polynomial (1) with matrix

$$C = \begin{bmatrix}
1 & + & + & + & + & + & + & + \\
+ & 2 & + & 3 & + & -1 & + & -1 \\
+ & + & 1 & + & + & + & + & + \\
+ & 3 & + & 3 & + & 4 & + & -2.8 \\
+ & + & + & + & 1 & + & + & + \\
+ & -1 & + & 4 & + & 2 & + & -1 \\
+ & + & + & + & + & 1 & + & + \\
+ & -1 & + & -2.8 & + & -1 & + & 3
\end{bmatrix} \in R_8,$$

where the symbol “+” denotes arbitrary nonnegative number, i.e., $c_i = c_i^+$, $i \in I_4 \equiv \{1, 3, 5, 7\}$, $c_{ii} = 1$, $i \in I_k$ and

$$p_1(\tilde{\alpha}) = \tilde{\alpha}^T C(4)\tilde{\alpha}, \tilde{\alpha}^T = (\alpha_2 \alpha_4 \alpha_6 \alpha_8), C(4) = \begin{bmatrix}
2 & 3 & -1 & -1 \\
3 & 3 & 4 & -2.8 \\
-1 & 4 & 2 & -1 \\
-1 & -2.8 & -1 & 3
\end{bmatrix} \not\in L(4).$$

The minimal eigenvalues of the matrices $C(4)$ and $C_0(4)$ are computed as $\lambda_{\text{min}}[C(4)] = -3.1732$, $\lambda_{\text{min}}[C_0(4)] = -0.5293$, which means that positivity of this polynomial cannot be concluded from these facts.
The two positive coefficients of the monomials $\alpha_2\alpha_4$ and $\alpha_4\alpha_6$, define the four subsets as follows:

$$
\Gamma_1 \equiv \{ \alpha_2 \leq \alpha_4, \ \alpha_4 \leq \alpha_6 \}, \quad \Gamma_2 \equiv \{ \alpha_2 > \alpha_4, \ \alpha_4 > \alpha_6 \},
$$

$$
\Gamma_3 \equiv \{ \alpha_2 \leq \alpha_4, \ \alpha_4 > \alpha_6 \}, \quad \Gamma_4 \equiv \{ \alpha_2 > \alpha_4, \ \alpha_4 \leq \alpha_6 \}.
$$

The respective matrices, corresponding to these subsets are easily computed from (3) as

$$
C_1(4) = \begin{bmatrix}
8 & 0 & -1 & -1 \\
0 & 11 & 0 & -2.8 \\
-1 & 0 & 2 & -1 \\
-1 & -2.8 & -1 & 3
\end{bmatrix}, \quad C_2(4) = \begin{bmatrix}
2 & 0 & -1 & -1 \\
0 & 9 & 0 & -2.8 \\
-1 & 0 & 10 & -1 \\
-1 & -2.8 & -1 & 3
\end{bmatrix},
$$

$$
C_3(4) = \begin{bmatrix}
8 & 0 & -1 & -1 \\
0 & 3 & 0 & -2.8 \\
-1 & 0 & 10 & -1 \\
-1 & -2.8 & -1 & 3
\end{bmatrix}, \quad C_4(4) = \begin{bmatrix}
2 & 0 & -1 & -1 \\
0 & 17 & 0 & -2.8 \\
-1 & 0 & 2 & -1 \\
-1 & -2.8 & -1 & 3
\end{bmatrix},
$$

where $\lambda_{\min}[C_1(4)] = 0.823$, $\lambda_{\min}[C_2(4)] = 0.804$, $\lambda_{\min}[C_3(4)] = 0.0694$, $\lambda_{\min}[C_4(4)] = 0.1583$, i.e., all matrices are positive definite and the considered polynomial is positive on the uncertainty set in accordance with Lemma 3.4.

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